

19/10/2016 الارباء

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حاضرة [4]

A comparison between binary and bipolar sigmoidal functions.

	Binary Sigmoid	Bipolar Sigmoid
Definition	$f(x) = \frac{1}{1 + e^{-x}}$	$g(x) = \frac{2}{1 + e^{-x}} - 1 = \frac{1 - e^{-x}}{1 + e^{-x}}$
Graph		
Range	$0 < f(x) < 1$	$-1 < g(x) < 1$
Relation between $f(x)$ and $g(x)$	$g(x) = 2f(x) - 1$	
Value of x	$x = \ln \left[\frac{f(x)}{1 - f(x)} \right]$	$x = \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]$
Derivative	$\begin{aligned} \frac{df(x)}{dx} &= \frac{e^{-x}}{(1 + e^{-x})^2} \\ &= f(x) [1 - f(x)] \end{aligned}$	$\begin{aligned} \frac{dg(x)}{dx} &= \frac{2e^{-x}}{(1 + e^{-x})^2} \\ &= 0.5 [1 - g^2(x)] \end{aligned}$
Graph of derivative		

Range of values of derivative	$0 < \frac{df(x)}{dx} < 0.25$	$0 < \frac{dg(x)}{dx} \leq 0.5$
Relation between $\frac{df(x)}{dx}$ and $\frac{dg(x)}{dx}$	$\frac{dg(x)}{dx} = 2 \frac{df(x)}{dx}$	

generalization $\Rightarrow f(x) = \frac{1}{1 + e^{-\alpha x}}$; $\alpha \equiv$ positive Parameter

* α adds another degree of freedom alongside the weights (a new parameter)

sigmoidal functions of the forms

$$(1) f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$(2) g(x) = \frac{2}{1 + e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$(3) h(x) = \tanh(\alpha x) \text{ where } \alpha \equiv \text{Positive Parameter}$$

* $f(x) = \frac{1}{1 + e^{-\alpha x}}$; verify that

$$x = \frac{1}{\alpha} \ln \left[\frac{f(x)}{1 - f(x)} \right] \quad \text{نصف الوتة، الوتة العكسية} \quad \alpha = 1 \text{ mV}^{-1}$$

→ Proof

$$f(x) + f(x) e^{-\alpha x} = 1$$

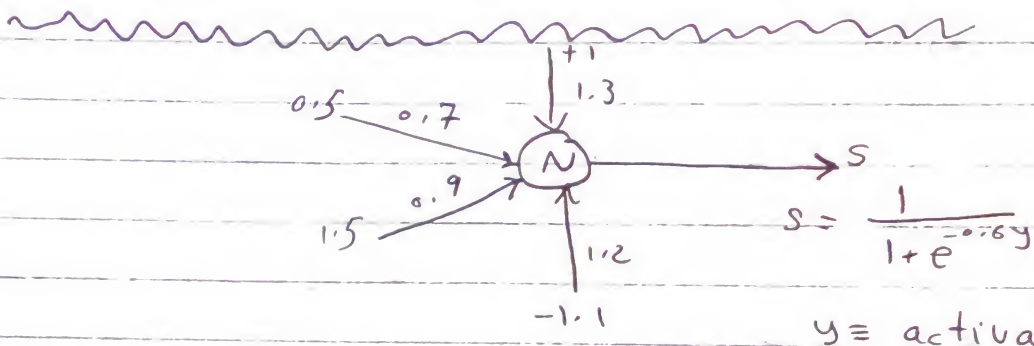
$$e^{-\alpha x} = \frac{1 - f(x)}{f(x)}$$

$$-\alpha x = \ln \left[\frac{1 - f(x)}{f(x)} \right]$$

$$\alpha x = \ln \left[\frac{f(x)}{1 - f(x)} \right]$$

$$x = \frac{1}{\alpha} \ln \left[\frac{f(x)}{1 - f(x)} \right]$$

$$\text{AND } \alpha = \frac{1}{x} \ln \left[\frac{f(x)}{1 - f(x)} \right]$$



$y \equiv \text{activation}$

$$\ast \text{ Activation } y = (0.5)(0.6) + (0.7)(1.1) + (0.9)(1.3) + (1.5)(-1.1) + (1.3) = 1.68$$

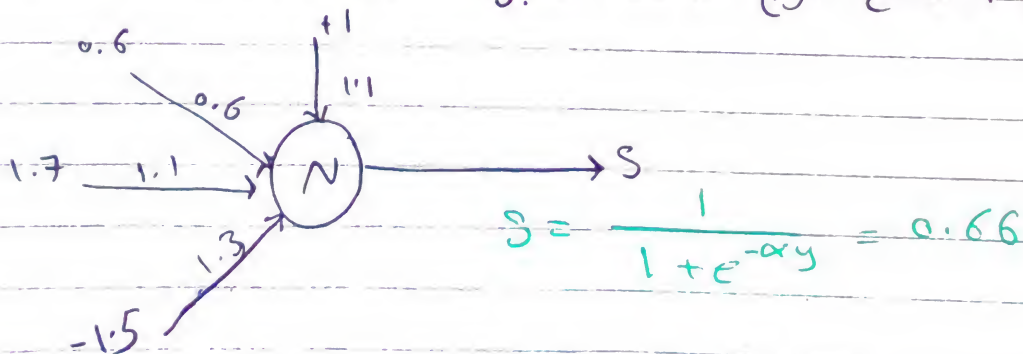
$$S = \frac{1}{1 + e^{-0.6(1.68)}} = 0.733$$

ليقبل Neuron عدد ثلاث 0.6، 1.1، 1.3 و 0.6 و 1.7 و -1.5

$y \equiv$ is activation fn

$\alpha \equiv$ positive parameter

أو قيمة α لتقريب المزج Bias = 1.1



Activation

$$y = (0.6)(0.6) + (1.7)(1.1) + (1.5)(1.3) + 1.1$$
$$= 1.38$$

$$\alpha = \frac{1}{y} \ln \left[\frac{f(y)}{1-f(y)} \right] = \frac{1}{y} \ln \left[\frac{s}{1-s} \right]$$
$$= \frac{1}{1.38} \ln \left[\frac{0.66}{1-0.66} \right] = 0.481$$

* Differentiation

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that

$$\frac{d f(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$
$$= \alpha f(x) [1 - f(x)]$$

تفسير شكل الجدول $\alpha = 1$

Proof

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$\frac{d f(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2} \quad (\text{first form})$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \cdot \frac{e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \cdot \frac{1 + e^{-\alpha x} - 1}{1 + e^{-\alpha x}}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \left[1 - \frac{1}{1 + e^{-\alpha x}} \right]$$

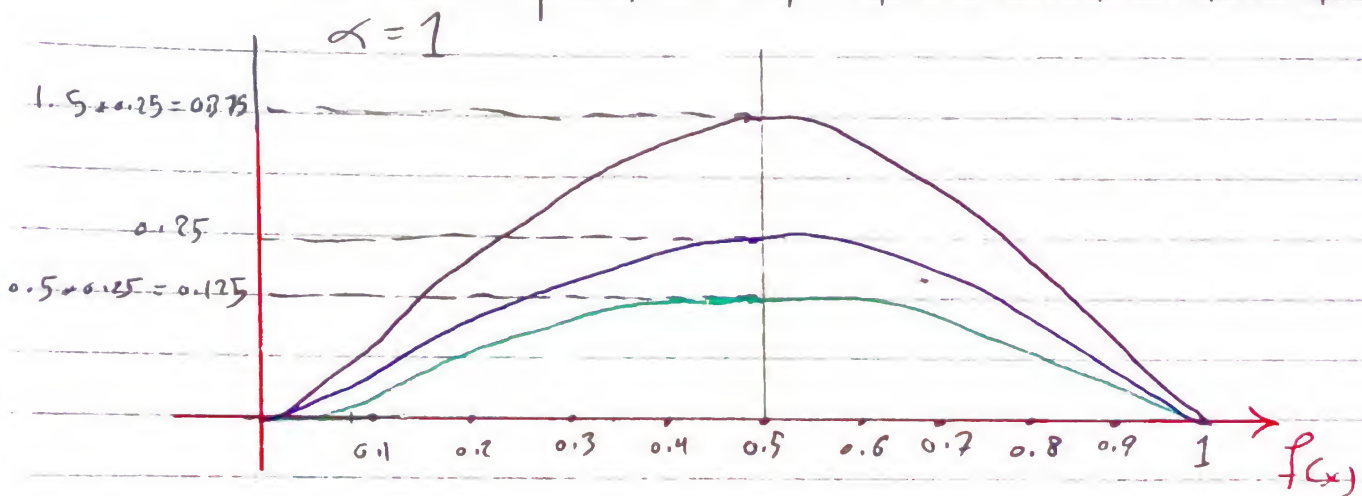
$$= \alpha f(x) [1 - f(x)]$$

ارسم شكل التفاضل $f(x)$ vs $\frac{df(x)}{dx}$ عند $\alpha = 0.5, 1, 1.5$

أثبت أن القيمة العظمى للتفاضل $\alpha \cdot 0.25$ وتحدث عند $f(x) = 1/2$

$$\frac{df(x)}{dx} = \alpha f(x) [1 - f(x)]$$

$f(x)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$df(x)/dx$	0	0.09				0.25				0.09	0



موضح من هذا شكل أن القيمة العظمى للتفاضل هي $\alpha \cdot 0.25$ وتحدث عند $f(x) = 0.5$

بعض النظر عن قيمة α فإن $f(x) = 0$ تعطي تفاضل بقيمة 0
 وكذلك $f(x) = 1$

يمكن إثبات هذه النتيجة رياضياً كما يلي

Mathematically,

$$\begin{aligned} \frac{df(x)}{dx} &= \alpha f(x) [1 - f(x)] \\ &= \alpha f(x) - \alpha f(x)^2 \end{aligned}$$

$$\frac{d \left[\frac{df(x)}{dx} \right]}{df(x)} = \alpha - 2\alpha f(x) = 0 \Rightarrow f(x) = 0.5$$

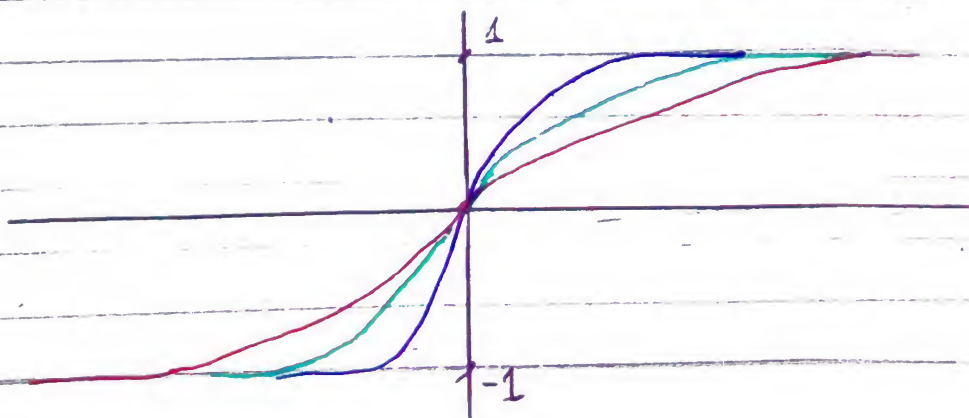
$$\left. \frac{d f(x)}{dx} \right|_{\max} = \alpha (0.5)(1-0.5) = 0.25 \alpha$$

فقد غلظت، اذ يمكن التنبؤ Max أو min. الثاني هو كما هو موجب، اذا هي min وعلى ذلك هو min

Generalization $\rightarrow g(x) = \frac{2}{1-e^{-\alpha x}} - 1 = \frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}$

- Draw on the same coordinates axes, the graphs of $g(x)$ for $\alpha = 0.5, 1$, and 2
- Comment on these graphs
- verify that $x = \frac{1}{\alpha} \ln \left[\frac{1+g(x)}{1-g(x)} \right]$
- verify that $\frac{dg(x)}{dx} = 0.5 \alpha [1-g^2(x)]$

x	$g(x), \alpha = 0.5$	$g(x), \alpha = 1$	$g(x), \alpha = 2$
-5	-0.848	-0.987	-0.999
-4	-0.762	-0.964	-0.998
-3			
-2			
-1			
0	0	0	0
1			
2			
3			
4	0.762	0.964	0.998
5	0.848	0.987	0.999



$\alpha = 1.5$

$\alpha = 1$

$\alpha = 0.5$

* مع زيادة قيم α فإنه الشكل البياني يقترب أكثر وأكثر من المحور الرأسي عند $x=0$ (بشكل متناظر)
 $\alpha \rightarrow \infty$ فإنه الدالة تصبح Bipolar threshold

$$\# g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$g(x) + e^{-\alpha x} g(x) = 1 - e^{-\alpha x}$$

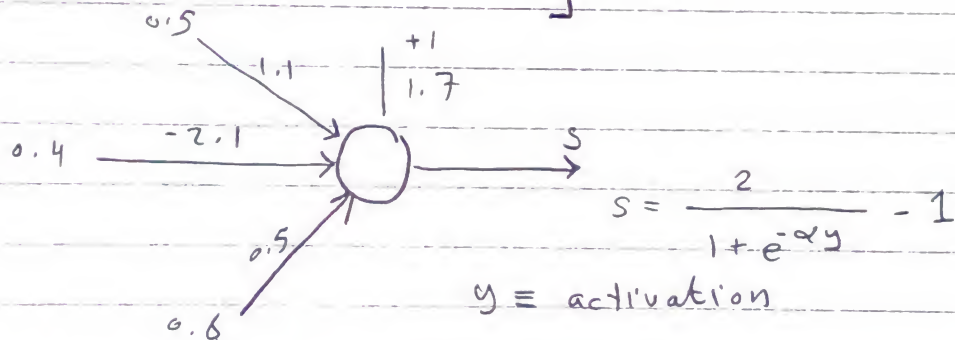
$$e^{-\alpha x} = \frac{1 - g(x)}{1 + g(x)} \Rightarrow -\alpha x = \ln \left[\frac{1 - g(x)}{1 + g(x)} \right]$$

$$\Rightarrow x = \frac{1}{\alpha} \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]$$

$$\# \frac{dg(x)}{dx} = \frac{(1 + e^{-\alpha x})(\alpha e^{-\alpha x}) - (1 - e^{-\alpha x})(-\alpha e^{-\alpha x})}{(1 + e^{-\alpha x})^2}$$

$$= 0.5 \alpha \left[1 - \left(\frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}} \right)^2 \right]$$

$$0.5 \alpha [1 - g^2(x)]$$



Find the value of the parameter α such that

$$S = 0.75$$

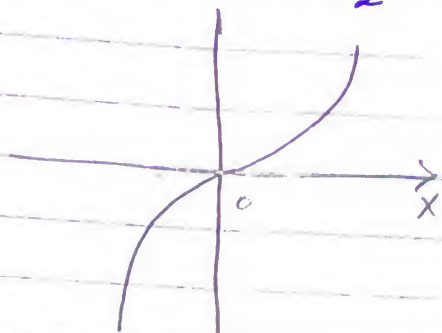
Activation $y = (0.5)(1.1) + (0.4)(-2.1) + (0.6)(0.5) + 1.7 = 1.71$

$$y = \frac{1}{\alpha} \ln \frac{1+S}{1-S} \Rightarrow \alpha = \frac{1}{y} \ln \frac{1+S}{1-S}$$

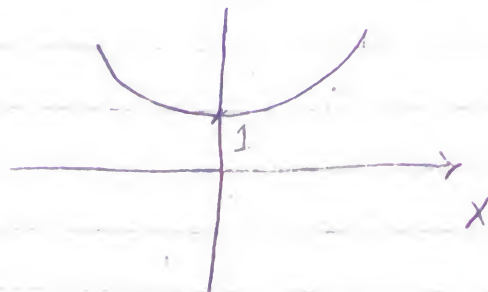
$$\alpha = \frac{1}{1.71} \ln \frac{1+0.75}{1-0.75} = 1.138$$

* Hyperbolic Functions

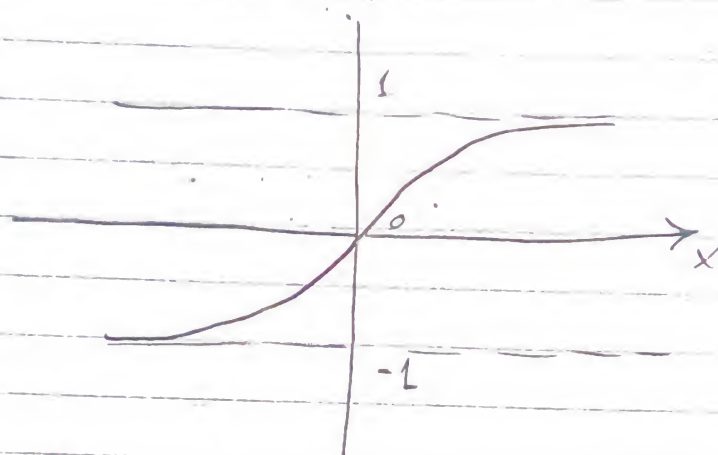
$$\sinh x = \frac{e^x - e^{-x}}{2}$$



$$\cosh x = \frac{e^x + e^{-x}}{2}$$



$$\tanh x = \frac{\sinh x}{\cosh x} \Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



$$\begin{aligned} \tanh x &= \frac{e^x \cdot e^{-x}}{e^x + e^{-x}} \\ &= \frac{1 - e^{-2x}}{1 + e^{-2x}} \end{aligned}$$

$$= \frac{2}{1 + e^{-2x}} - 1 = \text{bipolar Sigmoid at } \alpha = 2$$

$\tanh x = \text{bipolar Sigmoid at } \alpha = 2$

Generally,

$$\frac{2}{1 + e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}} \Rightarrow$$

$$= \frac{e^{\frac{\alpha x}{2}} - e^{-\frac{\alpha x}{2}}}{e^{\frac{\alpha x}{2}} + e^{-\frac{\alpha x}{2}}} = \tanh\left(\frac{\alpha x}{2}\right)$$

* وضع $z = \alpha$ نصل على النتيجة السابقة

$$\text{Bipolar Sigmoid} = \frac{2}{1 + e^{-\alpha x}} - 1 = \tanh\left(\frac{\alpha x}{2}\right)$$

نستخرج النتيجة السابقة \tanh Bipolar f_n